

Network-Constrained Energy Consumption Game for Dynamic Pricing Markets

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Abstract—The electricity distribution network is experiencing a profound transformation with the concept of the smart grid, providing possibilities for selfish consumers to interact with the distribution system operator (DSO) and to maximize their individual energy consumption utilities. However, this profit-seeking behavior among consumers may violate the network constraints, such as line flows, transformer capacity and bus voltage magnitude limits. Therefore, a network-constrained energy consumption (NCEC) game among active load aggregators (ALAs) is proposed to guarantee the safety of the distribution network. The temporal and spatial constraints of an ALA are both considered, which leads the formulated model to a generalized Nash equilibrium problem (GNEP). By resorting to a well-developed variational inequality (VI) theory, we study the existence of solutions to the NCEC game problem. Subsequently, a two-level distributed algorithm is proposed to find the variational equilibrium (VE), a fair and stable solution to the formulated game model. Finally, the effectiveness of the proposed game model and the efficiency of the distributed algorithm are tested on an IEEE-33 bus system.

Index Terms—Electricity distribution network, generalized Nash equilibrium problem, network-constrained energy consumption game, two-level distributed algorithm, variational equilibrium.

NOMENCLATURE

A. Indices and Sets

$b/\mathcal{B}$	Index/set of buses.
$l/\mathcal{L}$	Index/set of distribution lines.
$h/\mathcal{H}$	Index/set of time slots.
$m/\mathcal{M}$	Index/set of PLAs.
$n/\mathcal{N}$	Index/set of ALAs.
$\mathcal{B}_1(b)$	Set of all downstream buses of bus b .
$\mathcal{B}_2(b)$	Unique set of buses from the station bus to bus b .
$\mathcal{K}_n$	Set of deferrable cycles of ALA n .
$\mathcal{H}_{n,k}$	Set of time slots in deferrable cycle k of ALA n .
$s(l)/r(l)$	Indices of sending/receiving-end buses of distribution line l .

B. Parameters

R_l/X_l	Resistance/reactance of distribution line l .
v_1	Voltage magnitude at station bus.
$x_{n,h}^{\min}/x_{n,h}^{\max}$	Minimum/maximum energy consumption level of ALA n in time slot h .
$v_b^{\min}/v_b^{\max}$	Minimum/maximum allowed voltage magnitude.
$\omega_h/\gamma_{n,h}$	Utility parameter of ALA n in time slot h .
$\rho_h^{\min}$	Minimum retail price in time slot h made for ALAs by DSO.
η_h	Price sensitivity coefficient in time slot h made for ALAs by DSO.
$p_{m,h}^{\text{fix}}$	Energy consumption of PLA m in time slot h .
$s_l^{\max}$	Maximum complex power of line l .

C. Variables

$v_{b,h}$	Voltage magnitude at bus b in time slot h .
$x_{n,h}$	Active energy consumption of ALA n in time slot h .
$p_{l,h}^{\text{line}}/q_{l,h}^{\text{line}}$	Active/reactive line power of line l in time slot h .
$p_{b,h}^{\text{load}}/q_{b,h}^{\text{load}}$	Active/reactive load at bus b in time slot h .

In this list, only the main symbols used throughout the paper are given. Other symbols are abstract combinations of these symbols that are explained in the text. Bold lowercase symbols represent vectors and bold uppercase symbols represent matrices. $\mathbf{1}_N$ denotes the $N \times 1$ vector for all one's entries. $\mathbf{I}_N$ is the identity matrix of size N . $(\cdot)^T$ denotes the transpose of a vector or a matrix. Given N scalars x_n , where $n = 1, 2, \dots, N$, $[x_1, x_2, \dots, x_N]$ and $[x_1; x_2; \dots; x_N]$ represent the concatenation of all scalars by row and by column, respectively. For notation brevity, $[x_1; x_2; \dots; x_N]$ can be written as $[x_n]_{n=1}^N$. The vectors and matrices can also be concatenated by this notation if they have proper size. For a matrix $\mathbf{X}$, $[\mathbf{X}]_{(n,l)}$ extracts the entry in the n row and l column of $\mathbf{X}$. $\text{diag}(\mathbf{x})$ is a diagonal matrix with $\mathbf{x}$ as its diagonal. $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product. $\mathcal{X} \times \mathcal{Y}$ denotes the Cartesian product of two sets.

I. INTRODUCTION

ENERGY consumption around the world is rapidly on the rise due to the proliferation of intelligent applications, such as electric vehicles (EVs) [1] and data centers [2]. Without effective management, these new loads can easily cause peak loads and further threaten the safety of the distribution

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system [3]. In the past, the distribution system operator (DSO) usually addresses this problem by increasing the generation level and upgrading the network [4], which are both capital intensive. Nowadays, advances in smart grid technologies provide more choices for the DSO. An effective alternative is to implement demand management to alleviate the network congestion.

There are a lot of studies considering the network constraints in demand side management. These studies can be divided into two main categories. The first category in literature is concerned with power flow modeling and distributed optimization. A residential demand response with second-cone relaxation power flow model [5] was proposed in [6] to maximize the total utility of consumers, and a predictor correct proximal multiplier algorithm was used to safeguard the consumers' information. [7] extended the model presented in [6] to an unbalanced distribution network. In [8], a decentralized optimal dispatch of photovoltaic inverters with a semidefinite relaxation power flow model [9] was proposed. To accelerate the computing speed, a linearized power flow model was presented in [10]. The authors in [8] and [10] leveraged the alternating direction method of multipliers (ADMM) to obtain a distributed solution. [11] presented a penalty based ADMM approach for DC microgrids. These studies, however, assumed that all the flexible resources belonged to the DSO and did not mention to how to calculate the billing for each consumer or aggregator.

The second category in literature is concerned with using the distribution locational marginal price (DLMP) [12]–[18] to consider the impact of network constraints. [12] presented a DLMP framework: (i) the DSO calculated the DLMP and released this price signal to the aggregators; (ii) the aggregators scheduled their energy consumption according to the DLMP. [13] proposed a quadratic programming model to solve the problem of multiple solutions. [14], [15] used DLMP to dispatch the flexible buildings in congested distribution networks. However, [12]–[15] adopted a DC power flow model, in which only the line capacity was considered. Then, [16]–[18] attempted to investigate more accurate power flow models to derive DLMP. Although the DLMP is a dynamic pricing method for consumers, it only reflects the impact of network congestion on price. The consumers are still assumed to be the price-takers during the process of calculating the DLMP. In other words, the energy consumption of consumers will not change the retail price of the DSO.

With more consumers participating in demand response programs, the impact on the retail electric price of consumers' energy consumption must be considered. In addition, the consumers' selfish behaviors should also be included. [19] proposed a marginal pricing method for consumers. [20] additionally considered the power flow constraints and distributed generations based on the model proposed in [19]. Although this pricing method can achieve the individual interest maximization and social welfare interest maximization simultaneously, the consumers were charged more than the actual payment. [21] and [22] proposed a total energy consumption billing method, which is unfair for the consumers who used less electricity in peak times. [23]–[27] presented an instantane-

ous load billing method to charge the consumers. The linear energy price [23]–[25] and nonlinear energy price [26], [27] are both considered. These studies, however, assumed that the desired energy consumption of the consumers can always be met by the supply side.

There are also some studies trying to investigate the impact of the supply side. [28] and [29] considered the coupled capacity constraints for EV charging and the capital budget for energy trading, respectively. [30] considered the maximum supply ability of a provider and made sure that the total energy consumption of all consumers was lower than the maximum supply limit. [31] considered the lower and upper bounds on the aggregate load. [32] modeled the EV aggregator's capacity and renewable energy availability in an EV charging game. [33] studied the impact of limited capacities imposed by the electrical feeder and the gas pipelines. These studies assumed all agents were in the same location and only considered the capacity constraints.

Compared to previous studies, this paper adopts a clear, balanced, and fair billing method for consumers considering the network constraints imposed by the topology and operation of the distribution system. The formulated model is a network-constrained energy consumption (NCEC) game. In the proposed game model, active load aggregators (ALAs) act on behalf of flexible consumers competing to maximize their individual energy consumption utilities, and the DSO is responsible for ensuring the final energy consumption is safe and fair. A distributed algorithm based on the variation inequality (VI) theory [34] is used to find one of the equilibria for the ALAs.

The main contributions of this paper are as follows:

The network constraints are considered in the framework of instantaneous load billing [23]–[33]. To model the network constraints in this game, we derive a closed form of line power and voltage magnitude with respect to the energy consumption of the ALAs. A special generalized Nash equilibrium (GNE), denoted by variational equilibrium (VE), is used to fairly allocate the network resources.

A two-level distributed algorithm is designed to find the VE of the formulated game model, so the privacies of ALAs can be protected. The DSO first updates his control strategies at the upper level, and the ALAs successively update their energy consumption strategies at the lower level. The DSO is only assigned to evaluate an inexpensive linear algebraic term and is in charge of data collections and distributions.

The proposed model and algorithms are tested on an IEEE 33 bus system with seven ALAs. The proposed model can effectively alleviate the network congestion and the proposed algorithm can find a high precision solution within one minute. Therefore, this model can be ready for real-time applications.

II. SYSTEM MODEL

Consider a distribution system with a set of $\mathcal{B} := \{1, 2, \dots, B\}$ buses and a set of $\mathcal{L} := \{1, 2, \dots, L\}$ distribution lines. The bus 1 is the station bus where the DSO is located and the others are load buses. The consumers meet their energy consumption needs by procuring the energy from the DSO.

Such purchases can be done through a load aggregator (LA) on behalf of some consumers. According to the consumers' willingness to alter their energy consumption, the LAs can be divided into active load aggregators (ALAs) and passive load aggregators (PLAs). The energy consumption of a PLA is fixed while energy consumption of an ALA can be shifted and reduced. Let $\mathcal{N} := \{1, 2, \dots, N\}$ and $\mathcal{M} := \{1, 2, \dots, M\}$ denote the set of ALAs and PLAs. Without loss of generality, we assume that each bus has *exactly* one LA, therefore we can obtain $B = M + N - 1$. The DSO and LAs use a two-way communication infrastructure to exchange the necessary information, as shown in Fig. 1. In addition, we assume that the cycle of day is divided into a set of $\mathcal{H} := \{1, 2, \dots, H\}$ time slots.

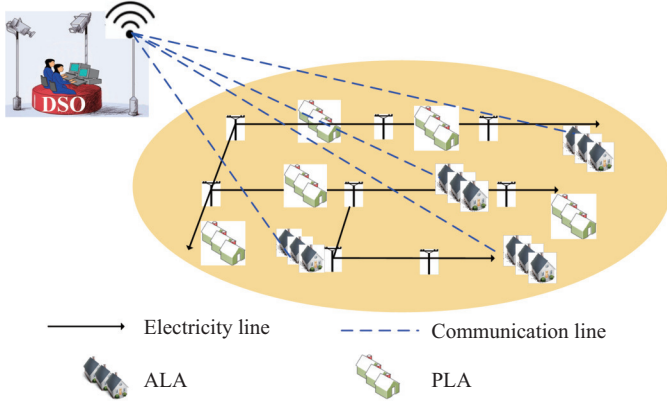


Fig. 1. A distribution network consisting of a DSO, several ALAs, and several PLAs.

A. Power Network Model

A distribution network is normally operated as a radial-like topology. Figure 2 shows a one-line diagram of a radial distribution power network. As customary, the voltage magnitude of the station bus v_1 is assumed to be given and fixed. The power flow can be described as a set of *Linearized DistFlow Branch Equations* [35], [36], as shown in (1)–(3).

$$p_l^{\text{line}} = \sum_{b \in \mathcal{B}_1(r(l))} p_b^{\text{load}} + p_r^{\text{load}} \quad (1)$$

$$q_l^{\text{line}} = \sum_{b \in \mathcal{B}_1(r(l))} q_b^{\text{load}} + q_r^{\text{load}} \quad (2)$$

$$v_b = v_1 - \sum_{r(l) \in \mathcal{B}_2(b)} (R_l p_l^{\text{line}} + X_l q_l^{\text{line}}) / v_1 \quad (3)$$

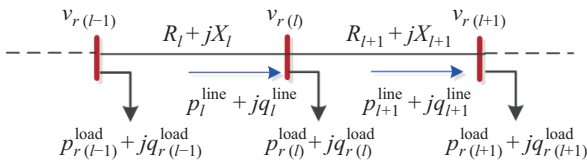


Fig. 2. One-line diagram of a radial power network.

For notation brevity, we define the four matrices $\mathbf{A} \in \mathbb{R}^{L \times B}$, $\mathbf{B}^x \in \mathbb{R}^{L \times L}$, $\mathbf{B}^r \in \mathbb{R}^{L \times L}$ and $\mathbf{C} \in \mathbb{R}^{B \times L}$, as shown

in (4)–(7).

$$[\mathbf{A}]_{(l,b)} = \begin{cases} 1 & b \in \mathcal{B}_1(r(l)) \text{ or } b = r(l) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$\mathbf{B}^x = \text{diag}([X_l]_{l=1}^L) \quad (5)$$

$$\mathbf{B}^r = \text{diag}([R_l]_{l=1}^L) \quad (6)$$

$$[\mathbf{C}]_{(b,l)} = \begin{cases} 1 & r(l) \in \mathcal{B}_2(b) \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Using the matrices defined above, (1)–(3) can be rewritten as follows:

$$\begin{bmatrix} \mathbf{p}^{\text{line}} \\ \mathbf{q}^{\text{line}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{p}^{\text{load}} \\ \mathbf{q}^{\text{load}} \end{bmatrix} \quad (8)$$

$$\mathbf{v} = v_1 \mathbf{1}_B - \frac{1}{v_1} \mathbf{C} (\mathbf{B}^r \mathbf{p}^{\text{line}} + \mathbf{B}^x \mathbf{q}^{\text{line}}) \quad (9)$$

where $\mathbf{p}^{\text{load}} = [p_b^{\text{load}}]_{b=1}^B$, $\mathbf{p}^{\text{line}} = [p_l^{\text{line}}]_{l=1}^L$, and $\mathbf{v} = [v_b]_{b=1}^B$.

Substituting (8) into (9), we can obtain:

$$\mathbf{v} = \boldsymbol{\kappa}^v + \mathbf{E}^p \mathbf{p}^{\text{load}} + \mathbf{E}^q \mathbf{q}^{\text{load}} \quad (10)$$

where $\boldsymbol{\kappa}^v = v_1 \mathbf{1}_B$, $\mathbf{E}^p = -\mathbf{C} \mathbf{B}^r \mathbf{A} / v_1$, and $\mathbf{E}^q = -\mathbf{C} \mathbf{B}^x \mathbf{A} / v_1$.

Without loss of generality, it is assumed that ratio φ_b between the reactive load q_b^{load} and active load p_b^{load} maintains a constant [37], so (10) becomes:

$$\mathbf{v} = \boldsymbol{\kappa}^v + \mathbf{E}^p \mathbf{p}^{\text{load}} \quad (11)$$

where $\mathbf{E} = \mathbf{E}^p + \mathbf{E}^q \text{diag}([\varphi_b]_{b=1}^B) \in \mathbb{R}^{B \times B}$.

B. Load Aggregator Model

The energy consumption of ALA n at time slot h is restricted by (12).

$$\mathcal{X}_n^1 = \{x_n | x_{n,h}^{\min} \leq x_{n,h} \leq x_{n,h}^{\max} \forall h \in \mathcal{H}\} \quad (12)$$

where $\mathbf{x}_n = [x_{n,h}]_{h=1}^H$ is the energy consumption vector of ALA n at time slot h .

Since some works must be fulfilled within a certain time range [24], [38], the total energy consumption during these periods should be larger than a given value. For example, an EV is connected to the grid at hour 14 and ready to leave at hour 18, so the total charging demand should be satisfied between the hours 14 and 18. Although we adopt an aggregate load model rather than a specific appliance model in this paper, this constraint may still exist in actual operations. This special energy consumption requirement is called as a deferrable cycle. Let $\mathcal{K}_n = \{1, 2, \dots, K_n\}$ denote the set of deferrable cycles of ALA n and $\mathcal{H}_{n,k} \subseteq \mathcal{H}$ denote the involved time slots. (13) describes this typical energy consumption requirement of ALA n .

$$\mathcal{X}_n^2 = \left\{ \mathbf{x}_n \left| \sum_{h \in \mathcal{H}_{n,k}} x_{n,h} \geq e_{n,k}^{\min} \forall k \in \mathcal{K}_n \right. \right\} \quad (13)$$

The utility function is used to measure the energy consumption satisfaction level. It is generally a non-decreasing and concave function with respect to the energy consumption. In

this paper, we choose the quadratic utility function [33], as shown in (14).

$$U_{n,h}(x_{n,h}) = \omega_h x_{n,h} - \frac{\gamma_{n,h}}{2} x_{n,h}^2 \quad (14)$$

where ω_h is a predetermined positive time-relative parameter, and $\gamma_{n,h}$ is a preference parameter for characterizing the consumption type with different ALAs and time slots.

C. Distribution Market

The DSO makes an appropriate pricing scheme for all LAs to obtain their profit. To distinguish the two types of LAs, different pricing schemes are designed. For the PLAs, a *fixed pricing* is typically adopted where the retail price is fixed at every time slot. A *dynamic pricing* is made for the ALAs where the retail price is related to the total energy consumption of the ALAs. A pricing scheme where the fixed price for the PLAs at a time slot is higher than the maximum dynamic price for the ALAs can motivate more consumers to participate in load management. However, how to make an efficient and fair pricing scheme for different types of LAs is beyond the scope of this paper. We adopt the *linear dynamic pricing* applied in [23]–[25] for ALAs.

$$\rho_h = \rho_h^{\min} + \eta_h \sum_{n \in \mathcal{N}} x_{n,h} \quad (15)$$

When LAs purchase electricity from the distribution market, a secure operation should be guaranteed, including the voltage magnitude regulation and power flow limit constraints, as given in (16) and (17).

$$\mathbf{v}^{\min} \leq \mathbf{v}_h \leq \mathbf{v}^{\max} \quad \forall h \in \mathcal{H} \quad (16)$$

$$(p_{l,h}^{\text{line}})^2 + (q_{l,h}^{\text{line}})^2 \leq (s_l^{\max})^2 \quad \forall l \in \mathcal{L}, h \in \mathcal{H} \quad (17)$$

where $\mathbf{v}^{\min} = [v_b^{\min}]_{b=1}^B$, $\mathbf{v}^{\max} = [v_b^{\max}]_{b=1}^B$, and $\mathbf{v}_h = [v_{b,h}]_{b=1}^B$. Note that the main transformer capacity can be implicitly included in constraint (17).

The voltage magnitude and line flow are related to the location of the ALs. For ease of presentation, the ALA connecting matrix $\mathbf{D}^a \in \mathbb{R}^{B \times N}$ and the PLA connecting matrix $\mathbf{D}^p \in \mathbb{R}^{B \times M}$ are introduced. The $\mathbf{D}^a$ can be defined with its $(b, n)^{\text{th}}$ element as 1 if ALA n is located at bus b and 0 otherwise. The $\mathbf{D}^p$ can also be defined in the same way. Let $\mathbf{p}_h^{\text{load}} = [p_{b,h}^{\text{load}}]_{b=1}^B$ denote the bus load column vector at time slot h . The $\mathbf{p}_h^{\text{load}}$ can be calculated by the following equation:

$$\mathbf{p}_h^{\text{load}} = \mathbf{D}^a \mathbf{x}_h + \mathbf{D}^p \mathbf{p}_h^{\text{fix}} \quad (18)$$

where $\mathbf{x}_h = [x_{n,h}]_{n=1}^N$, and $\mathbf{p}_h^{\text{fix}} = [p_{m,h}^{\text{fix}}]_{m=1}^M$.

Combining (11), (16), and (18), we can obtain:

$$\mathbf{m}_h^{\min} \leq \mathbf{M} \mathbf{x}_h \leq \mathbf{m}_h^{\max} \quad \forall h \in \mathcal{H} \quad (19)$$

where $\mathbf{M} = \mathbf{E} \mathbf{D}^a$, $\mathbf{m}_h^{\min} = \mathbf{v}^{\min} - \boldsymbol{\kappa}^v - \mathbf{E} \mathbf{D}^p \mathbf{p}_h^{\text{fix}}$, and $\mathbf{m}_h^{\max} = \mathbf{v}^{\max} - \boldsymbol{\kappa}^v - \mathbf{E} \mathbf{D}^p \mathbf{p}_h^{\text{fix}}$.

Although (17) is a convex set, it is nonlinear which can be unfavorable to the optimization. A regular S -sided polygon [39] is applied to linearize it.

$$\mathbf{p}_h^{\text{line}} \cos(s\alpha) + \mathbf{q}_h^{\text{line}} \sin(s\alpha) \leq \text{diag}([\beta_l]_{l=1}^L) \mathbf{s}^{\max} \quad \forall h \in \mathcal{H}, s \in \mathcal{S} \quad (20)$$

where $\alpha = 2\pi/S$, $\mathbf{p}_h^{\text{line}} = [p_{l,h}^{\text{line}}]_{l=1}^L$, $\mathbf{s}^{\max} = [s_l^{\max}]_{l=1}^L$, $\mathcal{S} := \{1, 2, \dots, S\}$. $\beta_l \in [\cos(\alpha/2), 1]$ is a conservative parameter, $\beta_l = \cos(\alpha/2)$ denotes the whole regular polygon falling into the circle. $\beta_l = 1$ denotes that the whole circle lies within the regular polygon.

Define $\mathbf{z}^p = [\cos(s\alpha)]_{s=1}^S$ and $\mathbf{z}^q = [\sin(s\alpha)]_{s=1}^S$, (20) can be rewritten as:

$$\mathbf{Z} \mathbf{p}_h^{\text{load}} \leq \mathbf{z}^{\max} \quad \forall h \in \mathcal{H} \quad (21)$$

where $\mathbf{z}^{\max} = \text{diag}([\beta_l]_{l=1}^L) \mathbf{s}^{\max} \otimes \mathbf{1}_S \in \mathbb{R}^{L \cdot S}$, and $\mathbf{Z} = (\mathbf{I}_L \otimes \mathbf{z}^p) \mathbf{A} + (\mathbf{I}_L \otimes \mathbf{z}^q) \mathbf{A} \text{diag}([\varphi_b]_{b=1}^B) \in \mathbb{R}^{(L \cdot S) \times B}$.

Substituting (18) into (21), we can obtain:

$$\mathbf{N} \mathbf{x}_h \leq \mathbf{n}_h^{\max} \quad \forall h \in \mathcal{H} \quad (22)$$

where $\mathbf{N} = \mathbf{Z} \mathbf{D}^a \in \mathbb{R}^{(L \cdot S) \times N}$, and $\mathbf{n}_h^{\max} = \mathbf{z}^{\max} - \mathbf{Z} \mathbf{D}^p \mathbf{p}_h^{\text{fix}} \in \mathbb{R}^{L \cdot S}$.

Remark 1: Since there is no distributed generation or storage system in this paper, the direction of line power is fixed. The effective feasible region of (17) is in the first quadrant. Thus, $\alpha = \pi/(2S)$ can be used. Consequently, the invalid constraints can be removed.

We define a matrix $\mathbf{R} = [-\mathbf{M}; \mathbf{M}; \mathbf{N}] \otimes \mathbf{I}_H$ and a vector $\mathbf{r} = \sum_{h \in \mathcal{H}} [-\mathbf{m}_h^{\min}; \mathbf{m}_h^{\max}; \mathbf{n}_h^{\max}] \otimes \mathbf{e}_h$ to put (19) and (22) together, where $\mathbf{e}_h \in \mathbb{R}^H$ is defined that its h^{th} element is 1 and others are 0.

$$\mathcal{Y} = \{\mathbf{x} | \mathbf{R} \mathbf{x} \leq \mathbf{r}\} \quad (23)$$

where $\mathbf{x} = [\mathbf{x}_n]_{n=1}^N$.

III. NETWORK-CONSTRAINED ENERGY CONSUMPTION GAME

In this section, we formulate the NCEC game among ALAs. By resorting to VI theory, we then investigate the existence of the GNE for the formulated game.

A. Game Formulation

Under the dynamic pricing scheme, the purchasing cost $C_{n,h}^{\text{pur}}$ of the ALA n at time slot h is given as follows:

$$C_{n,h}^{\text{pur}} = \rho_h x_{n,h} = \eta_h x_{n,h}^2 + (\rho_h^{\min} + \eta_h \sigma_{n,h}(\mathbf{x}_{-n,h})) x_{n,h} \quad (24)$$

where $\sigma_{n,h}(\mathbf{x}_{-n,h}) = \mathbf{1}_{N-1}^T \mathbf{x}_{-n,h}$.

The aim of an ALA is to maximize his net utility, i.e. utility minus the purchasing cost. To facilitate the derivation, the negative net utility is used as the objective function. This objective function can be regarded as a loss cost. The loss cost of the ALA n is given in (25).

$$\begin{aligned} C_n(\mathbf{x}_n, \sigma_n(\mathbf{x}_{-n})) \\ &= \sum_{h \in \mathcal{H}} C_{n,h}(x_{n,h}, \sigma_{n,h}(\mathbf{x}_{-n,h})) \\ &= \sum_{h \in \mathcal{H}} C_{n,h}^{\text{pur}}(x_{n,h}, \sigma_{n,h}(\mathbf{x}_{-n,h})) - U_{n,h}(x_{n,h}) \end{aligned} \quad (25)$$

where $\sigma_n(\mathbf{x}_{-n}) = [\sigma_{n,h}(\mathbf{x}_{-n,h})]_{h=1}^H$.

For clarity, the constraints are classified into two types of constraint sets.

1) Individual Constraint Set

The constraints only involve the individual ALA's decision variables $\mathbf{x}_n$, denoted by $\mathcal{X}_n$.

$$\mathcal{X}_n = \mathcal{X}_n^1 \cap \mathcal{X}_n^2 \quad (26)$$

2) Coupling Constraint Set

The constraints are coupled by all the ALAs' decision variables, i.e., $\mathcal{Y} \subseteq \mathbb{R}^{N \cdot H}$, shown in (23).

The energy consumption strategy $\mathbf{x}_n$ of ALA n should meet both its individual constraint set $\mathcal{X}_n$ and the coupling constraint set $\mathcal{Y}$. The feasible strategy of $\mathbf{x}_n$ is denoted as $\mathcal{Q}_n(\mathbf{x}_{-n})$, which means that this set depends on the energy consumption strategies of other ALAs.

$$\mathcal{Q}_n(\mathbf{x}_{-n}) = \{\mathbf{x}_n | \mathbf{x}_n \in \mathcal{X}_n, (\mathbf{x}_n, \mathbf{x}_{-n}) \in \mathcal{Y}\} \quad (27)$$

B. Game Model and GNE

From (25)–(27), it indicates that the desired energy consumption of an ALA depends on how he and other ALAs schedule their energy consumption. Therefore, it naturally belongs to a game framework. The game elements are given as follows.

- *Players*: ALAs in set $\mathcal{N}$.
- *Strategies*: Each ALA n selects its energy consumption strategy $\mathbf{x}_n$ from its strategy set $\mathcal{Q}_n(\mathbf{x}_{-n})$ to minimize his loss cost.
- *Cost functions*: The loss cost is defined in (25).

In sum, this NCEC game model $\mathcal{G}$ is formulated as follows:

$$\begin{cases} \min_{\mathbf{x}_n} C_n(\mathbf{x}_n, \boldsymbol{\sigma}_n(\mathbf{x}_{-n})) \\ \text{s.t. } \mathbf{x}_n \in \mathcal{Q}_n(\mathbf{x}_{-n}) \end{cases} \quad \forall n \in \mathcal{N} \quad (28)$$

Obviously, the interactions among ALAs take place both at the level of objective functions and the level of constraint conditions. Therefore, this noncooperative game is indeed a GNEP, denoted by $\text{GNEP}(\mathcal{G})$.

Definition 1. A tuple of strategies $\mathbf{x}^* \in \mathbb{R}^{N \cdot H}$ is called a GNE, if and only if it satisfies the following inequalities [40]

$$\begin{aligned} C_n(\mathbf{x}_n^*, \boldsymbol{\sigma}_n(\mathbf{x}_{-n}^*)) &\leq C_n(\mathbf{x}_n, \boldsymbol{\sigma}_n(\mathbf{x}_{-n}^*)) \\ \forall \mathbf{x}_n \in \mathcal{Q}_n(\mathbf{x}_{-n}^*), \forall n \in \mathcal{N} \end{aligned} \quad (29)$$

The above definition means that no single ALA can benefit from a unilateral deviation from $\mathbf{x}_n^*$, if all other ALAs act according to it. The set of generalized Nash equilibria (GNEs) of the game $\mathcal{G}$ is denoted as $\text{GNE}(\mathcal{G})$.

In our formulation, every player shares the same coupling constraint set, and the constraints can be explicitly expressed by a series of inequalities, so it belongs to a *GNEP with convex shared constraints* [41].

C. Existence of the $\text{GNE}(\mathcal{G})$

Definition 2 ([34, Definition 1.1.1]). Given $\mathcal{C} \subseteq \mathbb{R}^{N \cdot H}$ and a mapping $\mathbf{G}(\cdot) : \mathcal{C} \rightarrow \mathbb{R}^{N \cdot H}$, the variational inequality (VI) problem, denoted $\text{VI}(\mathcal{C}, \mathbf{G})$, is to find a vector $\mathbf{x}^* \in \mathcal{C}$ such that:

$$(\mathbf{x} - \mathbf{x}^*)^T \mathbf{G}(\mathbf{x}^*) \geq 0 \quad \forall \mathbf{x} \in \mathcal{C} \quad (30)$$

The set of solutions to this problem is denoted $\text{SOL}(\mathcal{C}, \mathbf{G})$.

The gradient of the objective function of each ALA with respect to its own strategy is stacked for defining function $\mathbf{G}(\cdot)$. In addition, we define $\mathcal{X}_1 \times \dots \times \mathcal{X}_N = \mathcal{X} \subseteq \mathbb{R}^{N \cdot H}$ and $\mathcal{C} = \mathcal{X} \cap \mathcal{Y}$. Here, a $\text{VI}(\mathcal{C}, \mathbf{G})$ is defined as:

$$\mathbf{G}(\mathbf{x}) = [\partial_{\mathbf{x}_n} C_n(\mathbf{x}_n, \boldsymbol{\sigma}_n(\mathbf{x}_{-n}))]_{n=1}^N = \mathbf{F}\mathbf{x} + \mathbf{f} \quad (31)$$

where $\mathbf{F} = \text{diag}(\boldsymbol{\gamma}) + \mathbf{I}_N \otimes \text{diag}(\boldsymbol{\eta}) + \mathbf{1}_N \otimes (\mathbf{1}_N^T \otimes \text{diag}(\boldsymbol{\eta}))$, and $\mathbf{f} = \mathbf{1}_N \otimes (\boldsymbol{\rho}^{\min} - \boldsymbol{\omega})$.

Proposition 1. 1) Cost function $C_n(\mathbf{x}_n, \boldsymbol{\sigma}_n(\mathbf{x}_{-n}))$ of ALA n is twice continuously differentiable and convex in $\mathbf{x}_n$ for every fixed $\mathbf{x}_{-n}$; 2) the set $\mathcal{C}$ is non-empty, compact, and convex.

The proof of Proposition 1 is given in Appendix A.

Theorem 1. There exists at least a GNE for the NCEC game.

The proof is primarily based on the relationship between the $\text{GNEP}(\mathcal{G})$ and $\text{VI}(\mathcal{C}, \mathbf{G})$. By proving that $\text{SOL}(\mathcal{C}, \mathbf{G})$ is non-empty, we indirectly prove that there exists at least a GNE. The detailed proof of Theorem 1 is given in Appendix B.

Remark 2: For a Nash equilibrium problem (NEP), the uniqueness of the Nash equilibrium can be proved by the strictly convex objective function for each player [42]. However, a GNEP usually has multiple GNEs due to the coupling of strategy space.

IV. DISTRIBUTED ALGORITHMS FOR THE GAME

In fact, the equilibrium solution can be derived in a central manner by concatenating all ALAs' Karush-Kuhn-Tucker (KKT) conditions, given in (32).

$$\begin{cases} \nabla_{\mathbf{x}_n} L_n(\mathbf{x}_n, \boldsymbol{\sigma}_n(\mathbf{x}_{-n}), \boldsymbol{\lambda}_n) + \\ \mathcal{N}^0(\mathbf{x}_n, \mathcal{X}_n) \in \mathbf{0} \\ \mathbf{0} \leq \boldsymbol{\lambda}_n \perp \mathbf{R}\mathbf{x} - \mathbf{r} \leq \mathbf{0} \end{cases} \quad \forall n \in \mathcal{N} \quad (32)$$

where $\boldsymbol{\lambda}_n$ denotes the multipliers of the coupling constraints associated with ALA n . $\mathcal{N}^0(\mathbf{x}_n, \mathcal{X}_n) \{ \mathbf{d} | \mathbf{d}^T(\mathbf{y} - \mathbf{x}_n) \leq 0, \forall \mathbf{y} \in \mathcal{X}_n \}$ is the normal cone to $\mathcal{X}_n$ at $\mathbf{x}_n$.

Different values of $\boldsymbol{\lambda}_n$ can lead to different equilibrium solutions. In this paper, we are interested in finding a special solution of the NCEC game, where $\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2 = \dots = \boldsymbol{\lambda}_N = \boldsymbol{\lambda}$. This solution is also called a VE. Because a VE has the same Lagrange multiplier for all ALAs, it is a fairer solution without players' discrimination. The solutions of $\text{VI}(\mathcal{C}, \mathbf{G})$ are exactly $\text{VE}(\mathcal{G})$. Hereinafter, we use $\text{SOL}(\mathcal{C}, \mathbf{G})$ and $\text{VE}(\mathcal{G})$ interchangeably.

Although the equilibrium solution can be centrally solved by the DSO, two issues must now be carefully addressed. The ALAs should submit their all information to the DSO which invades their privacies, and the DSO will suffer a huge computational burden when the ALAs increase. A distributed control algorithm is urgently needed to tackle these two problems.

However, the global constraint set $\mathcal{Y}$ prevents a direct application of the distributed algorithm because $\mathcal{C}$ does not have a Cartesian structure. To this end, a new problem is

considered, which is presented in (33) and (34).

$$\begin{cases} \min_{\mathbf{x}_n} C_n(\mathbf{x}_n, \boldsymbol{\sigma}_n(\mathbf{x}_{-n})) + \boldsymbol{\lambda}^T(\mathbf{R}\mathbf{x} - \mathbf{r}) \\ \text{s.t. } \mathbf{x}_n \in \mathcal{X}_n \end{cases} \quad \forall n \in \mathcal{N} \quad (33)$$

and furthermore:

$$0 \leq \boldsymbol{\lambda} \perp \mathbf{R}\mathbf{x} - \mathbf{r} \leq 0 \quad (34)$$

where $\boldsymbol{\lambda} \in \mathcal{Z}$, $\mathcal{Z}$ is a nonnegative orthant defined as $\mathbb{R}^{2B \cdot H + L \cdot S \cdot H}$. The Lagrange multiplier $\boldsymbol{\lambda}$ can be interpreted as prices paid by the ALAs for violating the network constraints.

Lemma 1. If $\mathcal{C}$ satisfies Slater's constraint qualification, then $\hat{\mathbf{x}}$ is a VE($\mathcal{G}$) if and only if $(\hat{\mathbf{x}}, \hat{\boldsymbol{\lambda}})$ is a solution of (33) and (34).

Proof. It is proved by [41, Lemma 4.4]. Since the constraints in $\mathcal{C}$ are linear, the Slater's constraint qualification holds [43, Section 5.2].

When the multiplier $\boldsymbol{\lambda}$ is given, the (33) is a NEP problem, denoted by $\mathcal{G}_\lambda$. The game $\mathcal{G}_\lambda$ is associated with a new VI($\mathcal{X}, \bar{\mathbf{G}}$), with $\bar{\mathbf{G}}(\mathbf{x}) = \mathbf{G}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{R}$. For ease of presentation, we define a mapping: $\Phi(\cdot) : \mathcal{Z} \rightarrow \mathbb{R}^{N \cdot H}$. The solution of the game $\mathcal{G}_\lambda$, denoted $\mathbf{x}^*(\boldsymbol{\lambda})$, can be represented by the mapping $\mathbf{x}^*(\boldsymbol{\lambda}) = \Phi(\boldsymbol{\lambda})$. The $\mathbf{x}^*(\boldsymbol{\lambda})$ is unique due to the fact that $\bar{\mathbf{G}}$ is strongly monotone on $\mathcal{X}$, and can be calculated with Algorithm 1.

Algorithm 1 Jacobi Best response algorithm

Initialization: Set $\tau_1 = 0$. Each ALA n sets the initial energy consumption strategy $\mathbf{x}_n^{(0)}$ and acceptable residual $\varepsilon_n^{\text{ALA}}$. The parameter λ is given.

Step 1: The DSO collects the energy consumption strategies of all ALAs, and sends information $\sigma_n(\mathbf{x}_{-n}^{(\tau_1+1)})$ to each ALA.

Step 2: Each ALA n updates the $\mathbf{x}_n^{(\tau_1+1)}$ by (35) and calculates the residual $\varepsilon_{n,\tau_1}^{\text{ALA}} = \|\mathbf{x}_n^{(\tau_1+1)} - \mathbf{x}_n^{(\tau_1)}\|$.

$$\mathbf{x}_n^{(\tau_2+1)} = \min_{\mathbf{x}_n \in \mathcal{X}_n} C_n(\mathbf{x}_n, \sigma_n(\mathbf{x}_{-n}^{(\tau_1)})) + \boldsymbol{\lambda}^T [\mathbf{R}]_{(:,n)} \mathbf{x}_n$$

where $[\mathbf{R}]_{(:,n)}$ is a submatrix related to the ALA n .

Step 3: If $\varepsilon_{n,\tau_1}^{\text{ALA}} \leq \varepsilon_n^{\text{ALA}} \forall n \in \mathcal{N}$, terminate; Otherwise, proceed to **Step 1**.

Lemma 2. The problem (33)–(34) is equivalent to a nonlinear complementarity problem (35) with respect to $\boldsymbol{\lambda}$

$$0 \leq \boldsymbol{\lambda} \perp \mathbf{r} - \mathbf{R}\Phi(\boldsymbol{\lambda}) \geq 0 \quad (35)$$

The proof of Proposition 1 is given in Appendix B.

Lemma 3. The problem (35) is equivalent to a VI problem VI($\mathcal{Z}, \mathbf{r} - \mathbf{R}\Phi(\boldsymbol{\lambda})$).

Proof. Since the set $\mathcal{Z}$ is a cone, the two problems are equivalent according to [34, Proposition 1.1.3].

Combining the Lemma 2 and Lemma 3, we can obtain a meaningful framework. VI($\mathcal{Z}, \mathbf{r} - \mathbf{R}\Phi(\boldsymbol{\lambda})$) can be solved first to obtain an optimal solution $\boldsymbol{\lambda}^*$, and then the Algorithm 1 is used to calculate the final energy consumption strategies. Because the VI problem is usually solved in an iterative

manner, this will lead to a two-level algorithm: the DSO first updates the Lagrange multiplier $\boldsymbol{\lambda}$ at the upper level, then the ALAs successively update their energy consumption strategies until the best optimal solution is reached. The detailed algorithm framework is shown in Algorithm 2.

Algorithm 2 Two-level algorithm for the NCEC game

Initialization: Set $\tau_2 = 0$. The DSO also sets the initial strategy $\boldsymbol{\lambda}^{(0)}$ and acceptable error ε^{DSO} .

Step 1: DSO sends information $\boldsymbol{\lambda}^{(\tau_2)}$ to each ALA $n \in \mathcal{N}$.

Step 2: Run Algorithm 1, the DSO obtains the adjusted energy consumption $\Phi(\boldsymbol{\lambda}^{(\tau_2)})$.

Step 3: The DSO updates the $\boldsymbol{\lambda}^{(\tau_2+1)}$ by (47), and calculates the error $\varepsilon_{\tau_2}^{\text{DSO}} = \|\boldsymbol{\lambda}^{(\tau_2+1)} - \boldsymbol{\lambda}^{(\tau_2)}\|$

$$\boldsymbol{\lambda}^{(\tau_2+1)} = [\boldsymbol{\lambda}^{(\tau_2)} - \alpha(\mathbf{R}\Phi(\boldsymbol{\lambda}^{(\tau_2)}) - \mathbf{r})]^+ \quad (36)$$

where $\alpha > 0$ is a sufficiently small positive step-size, and $[\cdot]^+$ denotes the projection onto the nonnegative orthant.

Step 4: If $\varepsilon_{\tau_2}^{\text{DSO}} \leq \varepsilon^{\text{DSO}}$, terminate; Otherwise, set $\tau_2 = \tau_2 + 1$ and proceed to **Step 1**.

V. CASE STUDY

The case study is performed on an IEEE 33-bus distribution system with 12.66 KV nominal voltage, and the network parameters can be found in [35]. There are 7 ALAs located at buses 2, 8, 16, 21, 24, 27, and 32, and 25 PLAs located at the other buses, as shown in Fig. 3. The ratio between the reactive energy consumption and active energy consumption for all ALAs are set to 0.6. Other simulation data related to PLAs, such as the specific location and the percentage of active energy consumption, can be found in [44]. The voltage magnitude at station bus is set to 1.05 p.u., and the upper and lower bounds of voltage magnitude at all buses are set to 1.05 p.u. and 0.95 p.u., respectively.

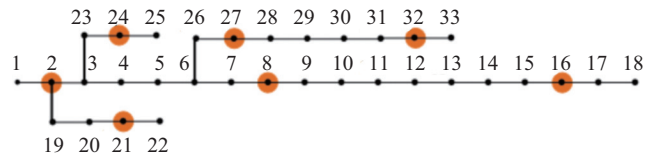


Fig. 3. IEEE 33-bus distribution system.

The normalized daily load profile is extracted from the UK national grid on June 15, 2020 [45]. The total energy consumption of all PALs at peak hour is 5.4 MW. The minimum and maximum energy consumption level of all ALAs at peak hour are set as 0.7 MW and 1.2 MW, respectively. The $\rho_h^{\min}$, η_h and ω_h are set to 200\$/MWh, 10\$/MWh, and 400\$/MWh at peak hour. The utility preference parameters $\gamma_{n,h}$ are chosen randomly from intervals $[100\$/(\text{MWh})^2, 140\$/(\text{MWh})^2]$ at peak hour. To investigate the effect of the location, we set the same preference parameters for ALA 2 and ALA 6, ALA 4 and ALA 5. Without loss of generality, these peak hour parameters are scaled to a multiperiod parameters by the normalized daily load profile. The deferrable characteristics of energy consumption for ALAs are shown in Table I.

TABLE I

DEFERRABLE CHARACTERISTICS OF ENERGY CONSUMPTION FOR ALAS

Index of ALA	Index of deferrable cycle	Time interval	Predetermined energy consumption (MWh)
1	1	[8, 12]	4.8
	2	[16, 20]	4.5
7	1	[1, 4]	2.0
	2	[12, 13]	2.0

The infinite norm is adopted for the convergence metric. We set $\varepsilon^{\text{DSO}} = 0.0005$, $\varepsilon_n^{\text{ALA}} = 0.1$ kW, and $\alpha = 0.0005$ to test the algorithm. The proposed model is coded in MATLAB with YALMIP [46] and solved with a Gurobi solver 9.0.3 [47]. All the tests are performed on a computer with a 3.41 GHz i7-6700 CPU and 8 GB memory.

The capacity of supply-side considered in [28]–[33] is actually the main transformer capacity. To make a detailed comparison, the main transformer capacity is only considered in this paper. Notably, the proposed model is universal for all capacity limits, including the main transformer capacity and distribution lines. The main transformer capacity is set to 13.5 MVA.

Four different scenarios are designed to examine the impact that the main transformer capacity and voltage magnitude limit have on ALA's decisions.

Scenario I: The main transformer capacity and voltage magnitude limits are both not considered.

Scenario II: Only the main transformer capacity is considered.

Scenario III: Only the voltage magnitude limit is considered.

Scenario IV: The main transformer capacity and voltage magnitude limits are both considered.

The system is under the most pressure at peak hour, and the system security deserves special attention. Therefore, we first focus our analysis on the operation at the peak hour without considering the deferrable characteristics.

A. The Operation At Peak Hour

Figure 4 and Table II give the voltage magnitude profiles of the distribution system and the total energy consumption of ALs in four different scenarios. As can be seen in Fig. 4, voltage magnitudes at some buses are below the lower bounds in both Scenario I and Scenario II, while the voltage magnitudes are all within the allowable range at all buses in Scenario IV and Scenario IV. The lowest voltage magnitude in Scenario I and Scenario II are 0.938 p.u. and 0.945 p.u., respectively. Although the voltage security is guaranteed in Scenario III, the total energy consumption exceeds the main transformer capacity, which is also in an insecure state. Thus, it is necessary to consider both the main transformer capacity and voltage magnitude limits for ensuring secure operations.

TABLE II

TOTAL ENERGY CONSUMPTION IN FOUR DIFFERENT SCENARIOS

Scenario	I	II	III	IV
Total energy consumption (MVA)	14.44	13.50	13.94	13.50

Figure 5 shows the energy consumption of seven ALAs at peak hour. In Scenario I, the preference parameters are the

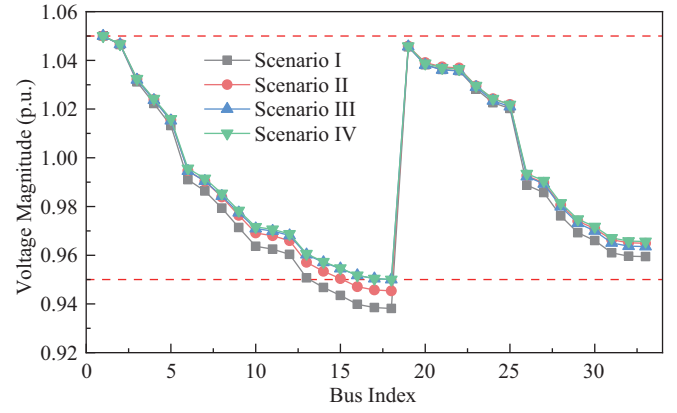


Fig. 4. Voltage magnitudes in four different scenarios.

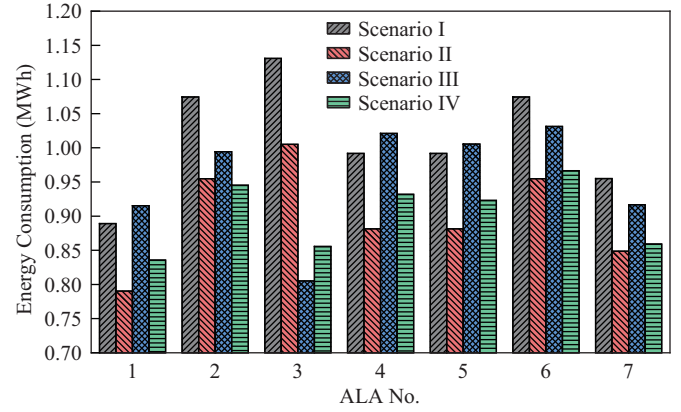


Fig. 5. Energy consumptions of seven ALAs at peak hour.

main factors to decide the energy consumption. A smaller $\gamma_{n,h}$ prefers to consume more energy to obtain the larger utility. When the main transformer capacity is considered (Scenario II), the energy consumption of all ALAs suffers different degrees of reductions. The energy consumption of ALA 1 is reduced from 0.89 MWh to 0.79 MWh. The energy consumption of ALA 2 and ALA 6 have the same reductions, about 0.12 MWh. This is due to the fact the ALA 2 and ALA 6 have the same preference parameter and contributions to the main transformer capacity. The similar phenomenon also happens to ALA 4 and ALA 5.

On the other hand, when the voltage magnitude limit is considered (Scenario III), not all ALAs tend to reduce energy consumption to guarantee secure operations. For example, ALA 1 is increased from 0.89 MWh to 0.92 MWh. The relative locations to the bus station and preference parameters can account for this interesting finding. Intuitively, since ALA 3 is far away from the bus station which has a significant effect on the voltage magnitude, ALA 3 has the largest reductions, about 0.33 MWh. Conversely, ALA 1 with a larger preference parameter is close to the substation, and the energy consumption is increased rather than reduced. The energy consumption of each ALA in Scenario IV is between the one in Scenario II and the one in Scenario III. This is primarily because Scenario II and Scenario III are two extreme scenarios compared with Scenario IV.

Table III lists the profit of each ALA in four different scenarios. A straightforward finding is that the profits of seven ALAs are not in line with the energy consumption of those in all scenarios. For Scenario I and II, more energy consumption will lead to a larger profit. However, this monotonous performance does not exist in Scenario III and IV. An interesting observation is that the total profit is increased when more constraints are considered. The reason is that the proposed game model is not designed to optimize total welfare as most of the demand response literatures do. Our model is to consider competing effects among ALAs, thus it is not an efficient solution to the maximization of all the ALAs' welfare.

TABLE III
PROFITS IN FOUR DIFFERENT SCENARIOS

Profit (\$)	Scenario I	Scenario II	Scenario III	Scenario IV
ALA1	61.27	66.35	65.28	67.47
ALA2	75.03	80.94	77.97	80.67
ALA3	79.30	85.45	73.46	80.47
ALA4	68.85	74.40	73.35	75.67
ALA5	68.85	74.40	73.17	75.47
ALA6	75.03	80.94	78.78	81.26
ALA7	66.12	71.51	69.50	71.79
Total	494.45	511.52	534.01	532.80

The optimal objective function values on all ALAs' welfare maximization in four scenarios are all \$544.87. Therefore, a more rigorous operational condition in our game model may lead to a more efficient solution in terms of social welfare.

B. The Operation Considering Temporally Coupled Constraint

The energy consumption of ALA 1 and ALA 7 with and without the temporally coupled constraint are shown in Figs. 6 and 7, respectively. The green light area highlights the time period of coupling. Figure 8 shows the total energy consumption of the seven ALAs from hour 8 to hour 20 in two conditions.

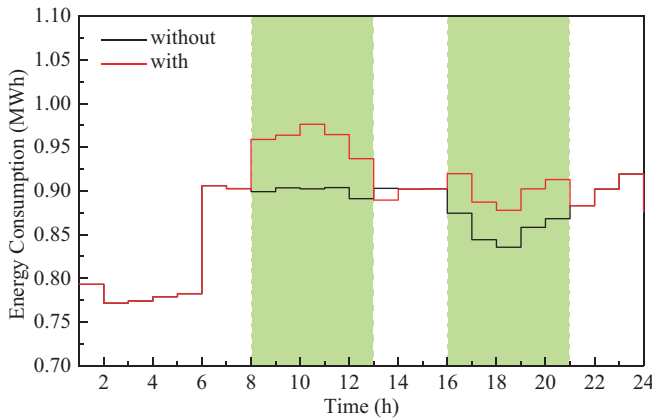


Fig. 6. Energy consumption of ALA 1 with and without temporally coupled constraint.

When the temporally coupled constraint is considered, the energy consumption of each individual ALA is very different from the one without temporally coupled constraint. Although each individual ALA has different energy consumption, the total energy consumption is almost the same under the two

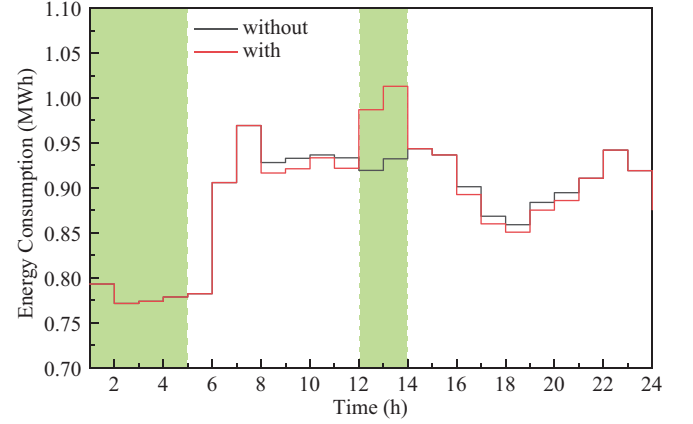


Fig. 7. Energy consumption of ALA 7 with and without temporally coupled constraint.

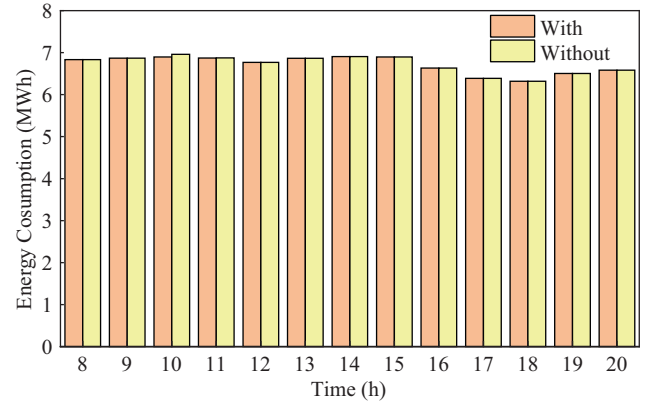


Fig. 8. Total energy consumption of seven ALAs with and without temporally coupled constraint.

conditions. This interesting observation is very meaningful. If the DSO only cares about his profit, he can implement the demand response programs without considering each ALA's temporally coupled constraint. This can be done in one slot by one slot and can greatly reduce the computing and communication burden. In other words, the temporally coupled constraint only changes the energy consumption of each individual ALA, but does not change the profit of the DSO.

C. Distributed Algorithm

Figure 9 plots the convergence of the two-level distributed algorithm. The DSO residual and ALA residual at iteration τ are defined as $\|\lambda^{(\tau)} - \lambda^{(\tau-1)}\|_\infty$ and $\max_{n \in \mathcal{N}} \|x_n^{(\tau)} - x_n^{(\tau-1)}\|_\infty$, respectively.

As can be seen, 37 DSO iterations and 178 ALA iterations are required to reach a converged solution. The reason why the residual of ALA is not monotonically decreasing is that a new NEP is conducted when the DSO changes its strategies. Hence, the number of peak points of the residual curve of ALA is equal to the number of DSO iterations.

An exact centralized solution can be obtained by solving (32). To check the exactness, the absolute error for all time periods of seven ALAs are calculated, as shown in Fig. 10. It can be shown that most of the absolute errors are lower than 0.001 MW. The maximum absolute error is 0.0024 MW, thus

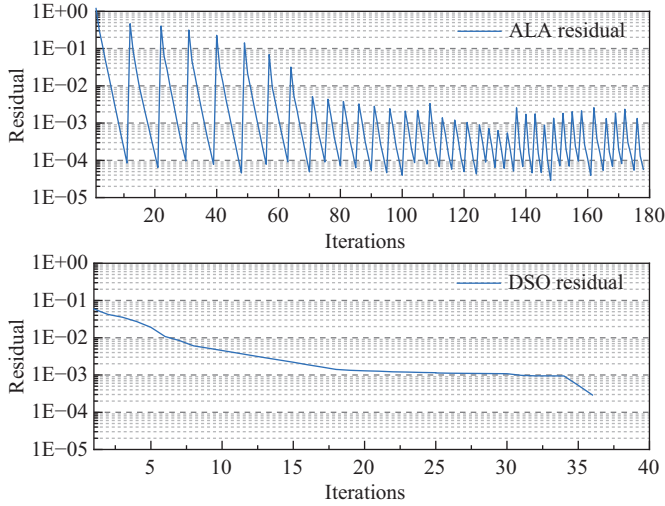


Fig. 9. Convergence of the two-level distributed algorithm.

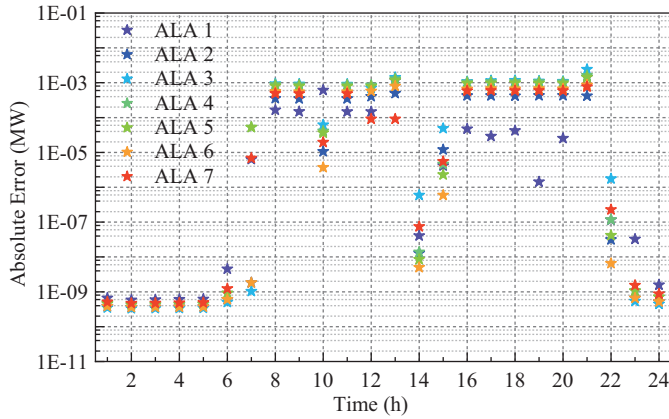


Fig. 10. Absolute error between a distributed solution and a centralized solution.

our proposed distributed algorithm is very exact.

To test the robustness of the algorithm, the step size α is set to different values. The results are listed in Table IV. To avoid over iterations, the maximum iteration of DSO is set to 200, and the maximum iteration of ALA for a given λ is set to 300. “Not Converged” means that the algorithm does not converge to a satisfactory solution within the maximum iterations. It can be concluded that a smaller α requires more iterations. But when the α is increased, the algorithm may not be converged. All converged cases can be finished within one minute. Therefore, the proposed distributed algorithm can be used for a real-time dispatch. In actual application, [0.001, 0.005] can be a better choice.

TABLE IV
COMPUTATIONAL PERFORMANCE FOR DIFFERENT STEP SIZES

α	DSO iterations	ALA iterations	Computation time (s)
0.00005	86	1446	8.24
0.0001	68	286	6.58
0.0005	37	178	4.83
0.001	Not converged	Not converged	/

VI. CONCLUSION

In this paper, an energy consumption game model considering the network constraints under dynamic pricing markets is proposed. Compared with the previous studies, the proposed model can guarantee the safety of the distribution network and fairly allocate the limited network resources. The ALA far away from the feeder tends to reduce more energy consumption, because it has the bigger impact on voltage violation. The DSO can obtain his profit one time slot by one time slot without considering the individual temporally coupled constraint of each ALA. In addition, the proposed two-level distributed algorithm can efficiently find the optimal solutions. In the future, this study can be extended in several directions. First, a lossless power flow model is used in this paper, and this physical grid model can be extended to consider network losses by using a more accurate set of power flow equations. Second, the distributed generation and storage can be incorporated to the proposed model for a bidirectional trading with the DSO.

APPENDIX

A. Proof of Proposition 1

1) Since the cost function is a quadratic function, the continuity and differentiability hold. The convexity can be proved by the second partials with respect to x_n .

$$\nabla_{x_n}^2 C_n(x_n, \sigma_n(x_{-n})) = \text{diag}(\gamma_n + 2\mu) > 0 \quad (A1)$$

Thus, the cost function is convex in x_n for every fixed x_{-n} .

2) The set $\mathcal{C}$ is expressed as sets of linear inequalities with finite upper and lower bounds, and thus convex and compact. The non-empty can be ensured by a practical problem. If the set $\mathcal{C}$ is empty, the current distribution network cannot satisfy the energy consumption requirement and should be upgraded.

B. Proof of Theorem 1

Because proposition 1 satisfies the assumption in [41, Lemma 4.2], every solution of $\text{VI}(\mathcal{C}, \mathbf{G})$ is also a solution of $\text{GNEP}(\mathcal{G})$. Furthermore, the $\mathcal{C} \subseteq \mathbb{R}^{N \cdot H}$ is convex and compact and mapping $\mathbf{G}(\cdot) : \mathcal{C} \rightarrow \mathbb{R}^{N \cdot H}$ is continuous, $\text{SOL}(\mathcal{C}, \mathbf{G})$ is nonempty and compact ([34 Corollary 2.2.5]).

C. Proof of Lemma 2

If the pair $(\hat{x}, \hat{\lambda})$ is a solution of the problem (33)–(34), then $\hat{x} = \Phi(\hat{\lambda})$ holds. The $\mathbf{0} \leq \hat{\lambda} \perp r - \mathbf{R}\Phi(\hat{\lambda}) \geq \mathbf{0}$ can be obtained by using (34). Thus, $\hat{\lambda}$ is also a solution of (35). If $\hat{\lambda}$ is a solution of the problem (36), the $\hat{x}$ can be obtained by the mapping $\Phi(\hat{\lambda})$, the pair $(\hat{x}, \hat{\lambda})$ is obviously a solution of the problem (33)–(34).

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